$\boxed{\text{Day 5 - AM}}$

Vectors

$$\begin{bmatrix} a \\ b \end{bmatrix} = (a,b) \in \mathbb{R}^2 \qquad \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3$$

A vector space is a nonempty set of vectors which are closed under addition and scalar multiplication.
↳ 10 axioms must hold for this to be true! (see textbook)

Linear Combination of vectors

$$\vec{y} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k \quad \text{where } c_i \in \mathbb{R},\ v_i \in \mathbb{R}^n$$

$\vec{y}$ is dependent on the $\vec{v}_i$'s if it is a linear combination of them

$\vec{y}$ is independent if not.

$\text{Span}\{\vec{v}_1, \dots, \vec{v}_k\}$ = set of all linear combinations of the $\vec{v}_i$'s

Ex:

$\text{Span}\{\vec{v}\} = \{\vec{u} : \vec{u} = t\vec{v} \text{ for some } t \in \mathbb{R}\}$
(line through the origin & $\vec{v}$)

$\text{Span}\{\vec{u}, \vec{v}\} = \{\vec{w} : \vec{w} = s\vec{u} + t\vec{v} \text{ for some } s,t \in \mathbb{R}\}$
(plane through origin, $\vec{u}$, and $\vec{v}$)

A subspace of a vector space V is a subset H of V such that
a) zero vector of V is in H
b) H is closed under vector addition
c) H is closed under scalar multiplication. →

Matrix Equations $A\vec{x}=\vec{b}$

$$\begin{bmatrix}\vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n\end{bmatrix}\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\\vdots\\b_m\end{bmatrix}$$

A is an
$m \times n$ matrix

$A\vec{x}=\vec{b}$ has a solution iff $\vec{b}\in \text{Span}\{\vec{a}_1,\dots,\vec{a}_n\}$
How do we find solutions? Row Reduce an Augmented Matrix!

Transformations: functions taking vectors to vectors
$T(\vec{x})=A\vec{x}$

Ex: ① projections $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)=\begin{bmatrix}x_1\\x_2\\0\end{bmatrix}$ $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$

② contraction/dilation $T(\vec{x})=r\vec{x}$
$0\le r\le 1$ $r>1$

③ reflection $A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

theres more.... rotations! shifts!

A transformation is linear if
$T(\vec{u}+\vec{v})=T(\vec{u})+T(\vec{v})$
$T(c\vec{u})=cT(\vec{u})$ for all $\vec{u},\vec{v}\in$ domain of T, $c\in\mathbb{R}$

Col A is the column space of A, the set of all linear combinations of the columns of A
ie: $\text{Span}\{\vec{a}_1,\vec{a}_2,\dots,\vec{a}_n\}$

→

Nul A denotes the nullspace of A, the solution set of $A\vec{x} = \vec{0}$